Problem Set 4

Problem 1. Equivalent Tax Systems

Solve Exercise 16.7-16.9 in Ljungqivst and Sargent.

Problem 2. The Problem of the Firms and Corporate Income Taxes In this problem, we try to write down a more realistic problem of the firm and use it to analyze coporate income taxes.

Consider the standard neoclassical model where there is a single final good per period that can be used for consumption and investment and that is produced with labor and capital. Production function is as usual given by $F(K, A_t N)$ where $A_t = (1+g)^t$ while capital depreciates at rate δ. Finally preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$$

which are the preferences of a unit continuum of consumers who are all identical – we have not yet described the endowments and how trading happens.

There is a representative firm that owns the stock of capital and its shares are traded by the households sequentially. In particular, in each period households can competitively trade shares of the representative firms. Each share of the firm entitles the households to the stream of dividends paid by the firm – note that we allow the firm to issue equity by setting dividends equal to a negative number. Households take as given the sequence of dividend payments.

The firm own its capital, purchases investment goods, hires workers and pays out dividends to households (potentially issuing equity by setting dividends equal to a negative number).

a. Write down the problem of the households assuming that they pay no form of taxes – there are no capital gains taxes and earnings taxes.

b. Write the optimality condition of households with respect to their holding of shares in each period. Using this Euler equation, calculate the price of the stock in each period as a funtion of the stream of dividends, and sequence of consumption.

c. Now suppose that there is a constant labor income tax, τ_l, a constant capital gains tax rate, τ_c and dividends are treated the same as labor income – for now suppose that government also subsidizes equity issuances at rate τ_c . Repeat part b.

d. Using the formula in part c to define the objective of the firm and write the optimization problem of the firm. Suppose that firms pay coporate income taxes, at rate τ_p, on profits before paying dividends – they can deduct cost of labor from their sales but can only deduct a fraction κ of their investment. Moreover, suppose that a fraction of depreciation, χ, can be deducted from the taxable corporate income.

e. Define a tax-distorted competitive equilibrium in this economy assuming that the government cannot issue debt.

f. Write down the equivalent of the inter- and intratemproal optimality conditions for this model with taxes. Determine how each element of the tax code (on the household and firm side) affect the invesment and labor decision.

h. Define a BGP for this economy. How does each policy instrument affect the BGP of this model.

g. Now suppose that firms can issue debt in addition to equity. We assume that debt is short-term and matures in one period. The interest paid on debt is deductible from the taxable income of firms. Moreover, the downside of debt issuance is a bankruptcy cost – which for now we assume it is exogenous and given by $\psi\left(b_{t}\right)$ – where $\psi\left(\cdot\right)$ is a stricly increasing and convex function with $\psi\left(0\right)=0$.[1] Note that this bankruptcy cost is not deductible. Suppose in addition that households purchase the debt issued by firms – in addition to their equity. Repeat the exercises b through h under this assumption.

h. Now suppose in addition to debt issued by firms, government can also issue debt. Redefine the tax-distorted competitive equilibrium . Does your answer to part g changes?

Problem 3. Optimal Taxes in the AKH model

Consider the AKH model (with cost of human capital accumulation in terms of the final good) that we discussed in class and assume that the government can impose taxes/subsidies on labor income, capital income and consumption of households. Recall that production function was Cobb-Douglas, preferences are given by $\sum_{t=0}^{\infty}\beta^{t}C_{t}^{1-\sigma}/\left(1-\sigma\right)$ and that depreciation of physical and human capital are the same.

a. Define a TDCE in this economy.

b. Show that an allocation is part of a TDCE if and only if it is feasible and satisfies an implementability constraint - derive the implementability constraint.

c. Are there any redundant taxes in this model?

d. Calculate optimal capital and labor income taxes.

e. Repeat part c and d assuming that labor supply is elastic and preferences are given by

$$\sum_{t=0}^{\infty}\beta^{t}\left[\frac{C_{t}^{1-\sigma}}{1-\sigma}-\frac{n_{t}^{1+1/\varepsilon}}{1+1/\varepsilon}\right]$$

where n_{t} is hours worked. Additionally, production function is given by

$$Y_{t}=AK_{t}^{\alpha}\left(H_{t}n_{t}\right)^{1-\alpha}.$$

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[1]Furthermore, $\psi\left(b\right)=\infty$ for all values of b that are negative.

What tax systems are general enough so that an implementability constraint is sufficient for characterization of a TDCE? What can you say about optimal taxes in the long-run?

Problem 4. Uniform Commodity Taxation - Non-Primal approach

In class, we discussed the problem of commodity taxes and characterized its solution using the primal approach. We further talked about the Diamond and Mirrlees's result on production efficiency. In this problem, you are solving it using the non-primal approach. Suppose that we have an economy where there are N consumption goods and a unit continuum of individuals have some wealth of the numeraire good, I. Suppose that the government imposes taxes on all consumption goods and cannot tax individuals' wealth. In addition, suppose that prices of all goods are given by $p_i = 1$, $i = 1, \cdots, N$ – this is without loss of generality. Finally suppose that the government has an expenditure G of the numeraire good.

a. For any tax schedule, τ_i, define after tax prices $q_i = 1 + \tau_i$. Formulate the optimal taxation problem using the indirect utility function, $V(\mathbf{q}; I)$ and consumer demand function

$$\mathbf{x}(\mathbf{q}; I) = (x_1(\mathbf{q}; I), \cdots, x_N(\mathbf{q}; I))$$

b. Using the formulation above, provide a formula for optimal taxes by manipulating the first order conditions of the above problem. Your formula should be a matrix equation in terms of the elasticity matrix (compensated or uncompensated) implied by the above demand functions.
Hint: Use Roy's identity!

c. Show that if utility of the individuals in the economy are given by $u(\mathbf{c}) = \left[\sum_{i=1}^{N} \alpha_i c_i^{1-\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$,

then optimal taxes over all of the goods are the same.
Hint: You can show this by showing that the constant taxes are a solution of the matrix equation in part b. Furthermore, show that the solution to this equation is unique.

d. What happens to optimal taxes when individuals are heterogeneous with respect to their wealth of the numeraire?

Problem 5. Optimal Policy in the Expanding Variety Model

Consider the expanding variety model of Romer and suppose that we introduced taxes into this model. Taxes can be time-dependent and on all margins of activity – saving, innovation, production of intermediate goods, and labor income. Suppose that government has access to debt.

a. Formulate this problem and define a TDCE.

b. Drive the implementability constraint.
Hint: This problem is different from the standard Ramsey problem discussed in class. What is the difference?

c. What can you say about optimal policy in this environment? How is it different than the standard Ramsey model's prescription?

Problem 6. Optimal Corporate Income Taxes
Consider the setting in part h of problem 2 and suppose that there is no debt. Furthermore, allow all policy variables to be time-dependent.

a. In this formulation, are there redundant taxes?

b. Drive the implementability constraint.

c. Given the tax instruments available to the government, is it possible to make implementability constraint non-binding? If so, what tax policies achieve this? What would be the prescription of this model on optimal coporate income taxes?

d. Suppose you were to use only taxes on households. Can you find a tax schedule that is equivalent to the optimal corporate income tax